Prediction of Total Phosphorus Concentrations, Chlorophyll *a*, and Secchi Depths in Natural and Artificial Lakes[1]

DANIEL E. CANFIELD JR.,[2] AND ROGER W. BACHMANN

Department of Animal Ecology, Iowa State University, Ames, IA 50011, USA

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A model for the prediction of total phosphorus was developed and tested using data on 704 nautral and artificial lakes including 626 lakes in the U.S. Environmental Protection Agency (EPA) National Eutrophication Survey. A statistical analysis showed that the best estimate for the sedimentation coefficient (σ) in the Vollenweider equation was

$\sigma = 0.162(L/z)^{0.458}$ for natural lakes and
$\sigma = 0.114(L/z)^{0.589}$

for artificial lakes where L is the areal phosphorus loading rate ($mg \cdot m^{-2} \cdot yr^{-1}$) and z is the mean depth (m). The model yields unbiased estimates of phosphorus concentrations over a wide range of lake types and has a 95% confidence interval of 31–288% of the calculated total phosphorus concentration. Other models are less precise. Though total phosphorus concentrations can be predicted equally well in natural and artificial lakes, predictions of algal densities and water transparency are less reliable in artificial lakes, as the phosphorus–chlorophyll and chlorophyll–Secchi depth relationships are less precise. This seems to be due to the influence of nonalgal particulate materials.

Key words: phosphorus models, eutrophication, lake trophic state

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Les données recueillies dans 704 lacs naturels et artificiels, y compris les 626 lacs inclus dans le relevé national de l'eutrophisation entrepris par l'Environmental Protection Agency (EPA) des É.-U. a servi à construire et à tester un modèle permettant de prédire le phosphore total. A l'analyse statistique, la meilleure estimation du coefficient de sédimentation (σ) dans l'équation de Vollenweider est:

$\sigma = 0{,}162(L/z)^{0{,}458}$ pour les lacs naturels et
$\sigma = 0{,}114(L/z)^{0{,}589}$

pour les lacs artificiels, où L est le taux de charge du phosphore par superficie ($mg \cdot m^{-2} \cdot an^{-1}$) et z la profondeur moyenne (m). Grâce à ce modèle, on obtient des estimations non biaisées des concentrations de phosphore dans une gamme étendue de types de lacs. Son intervalle de confiance à 95% est de 31 à 288% de la concentration de phosphore total calculée. Les autres modèles sont moins précis. On peut prédire les concentrations de phosphore total tout aussi bien dans les lacs naturels qu'artificiels. Cependant, les prédictions de densité des algues et de la transparence de l'eau sont moins fiables dans les lacs artificiels, car les relations phosphore–chlorophylle et chlorophylle–profondeur du disque de Secchi sont moins précises. Ceci semble dû aux matériaux particulaires autres que les algues.

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[2] Present address: School of Forest Resources and Conservation, University of Florida, Gainesville, FL 32611, USA.

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STUDIES have demonstrated that the concentration of total phosphorus in natural lakes can be an important indicator of lake trophic state (Vollenweider 1968; Dillon 1975), algal population densities as measured by chlorophyll *a* concentrations (Dillon and Rigler 1974a; Jones and Bachmann 1976), and water clarity (Bachmann and Jones 1974; Dillon and Rigler 1975). Recently, simple empirical models have been developed to predict lake total phosphorus concentrations from data on annual phosphorus inputs, hydraulic flushing rates, and lake morphometry (Vollenweider 1975; Kirchner and Dillon 1975; Chapra 1975; Jones and Bachmann 1976; Larsen and Mercier 1976; Reckhow 1977, 1979). Because these models require a minimum of data and the predicted phosphorus values can be used to estimate other accepted limnological measures of trophic state, lake managers and researchers are using them to predict the response of natural and artificial lakes to changes in phosphorus inputs (Chapra and Robertson 1977). However, the general application and potential limitations of these models and the relationship between total phosphorus concentrations and lake trophic state, chlorophyll *a* concentrations, and water clarity have not been thoroughly addressed.

The empirical phosphorus loading models have been developed from data on selected natural lakes in Canada, northern Europe, and the northern United States. Although the models seem to be different, they all are based upon the general model proposed by Vollenweider (1969):

$$(1)\quad TP = \frac{L}{\bar{z}(\sigma + \rho)}$$

where

TP = concentration of total phosphorus in the lake water, $\mathrm{mg \cdot m^{-3}}$
L = annual phosphorus loading per unit of lake surface area, $\mathrm{mg \cdot m^{-2} \cdot yr^{-1}}$
$\bar{z}$ = mean depth of the lake, m
σ = phosphorus sedimentation coefficient, $\mathrm{yr^{-1}}$
ρ = hydraulic flushing rate, $\mathrm{yr^{-1}}$.

The only substantial differences among the models are the forms of the empirical equations used to estimate phosphorus losses to the sediments, but these equations are critical to the successful prediction of lake phosphorus concentrations. Using a small sample of lakes, Vollenweider (1975) proposed that lake phosphorus sedimentation coefficients could be estimated by dividing 10 by lake mean depth. Jones and Bachmann (1976) showed that a constant phosphorus sedimentation coefficient of 0.65 $\mathrm{yr^{-1}}$ worked well for 51 natural lakes.

Other authors have chosen to reformulate the Vollenweider equation and work with the phosphorus retention coefficient of a lake (Dillon and Rigler 1974b) rather than with the phosphorus sedimentation coefficient:

$$(2)\quad TP = \frac{L(1-R)}{q_s}$$

where

q_s = annual areal water loading (lake outflow/lake surface area), $\mathrm{m \cdot yr^{-1}}$
R = phosphorus retention coefficient (difference between annual phosphorus inputs and phosphorus outputs divided by the annual phosphorus input).

Kirchner and Dillon (1975) developed a relationship between the phosphorus retention capacity and areal water loading for 15 southern Ontario lakes. Chapra (1975) used a different parameter, the apparent settling velocity (ν), equal to the product of the mean depth and the sedimentation coefficient. He showed that:

$$R = \frac{\nu}{\nu + q_s}$$

and took ν to be a constant. Reckhow (1979) subsequently proposed that ν varied with q_s.

$$\nu = 11.6 + 0.2q_s.$$

Larsen and Mercier (1976), by working with data from 20 lakes, suggested that phosphorus retention coefficients could be better estimated by the inverse of 1 plus the square root of the hydraulic flushing rate. Vollenweider (1976) independently obtained the same relationship.

Successful application of these empirical models is greatly dependent upon how well the models estimate phosphorus losses to the sediments. Though these models have been used with reasonable success on selected natural lakes (Jones and Bachmann 1976; Chapra and Robertson 1977), studies by Jones and Bachmann (1978) on several central Iowa artificial lakes have shown the direct use of these models overestimated summer phosphorus concentrations by 3 to 10 times. They suggest that this overestimation resulted because phosphorus sedimentation rates are greater in artificial lakes. By using sedimentation coefficients two orders of magnitude greater than those used for natural lakes, Jones and Bachmann (1978) were able to use the Vollenweider model (Eq. 1) to calculate summer phosphorus concentrations.

The importance of quantifying those factors controlling phosphorus sedimentation in natural and artificial lakes is evident (Eq. 1). This study was designed to determine the general applicability of empirical phosphorus loading models. Our approach was to examine the phosphorus input–output relationship of a large number of natural and artificial lakes to determine the general limnological factors that influence phosphorus sedimentation. By using these factors, we wished to determine if empirical models could be developed to predict total phosphorus concentrations in both natural and artificial lakes or if these two lake types needed to be treated differently. We also wished to test the predictive abilities of the new and published empirical phosphorus models and determine the degree to which total phosphorus concentrations can be used to predict chlorophyll *a* concentrations and Secchi disc depths in natural and artificial lakes.

Data Base

Data from a broad range of natural and artificial lakes were gathered from lakes cited in the published literature (Jones and Bachmann 1976; Larsen and Mercier 1976), the U.S.

Environmental Protection Agency National Eutrophication Survey (EPA–NES), and our unpublished studies in Iowa.

Data on annual areal phosphorus loading rates, lake mean depths, hydraulic flushing rates, and total phosphorus concentrations were tabulated for all lakes. We wanted the total phosphorus concentrations to be representative of the "average" conditions for the lake. For the EPA–NES lakes we used the median total phosphorus concentrations, agreeing with Reckhow (1977) that this value would be less affected by extreme measurements. For the lakes from the literature we used the values supplied by the authors, which were usually means. Chlorophyll *a* concentrations, Secchi disc transparencies, and total alkalinity values were tabulated for the EPA–NES lakes. Phosphorus sedimentation coefficients for each lake were estimated from the data by assuming steady state and rearranging the terms of Equation 1 (Jones and Bachmann 1976):

$$(3) \quad \sigma = \frac{L/z}{TP} - \rho.$$

The published literature provided data from 77 natural lakes located in Canada, northern Europe, and the northern United States. Our studies provided data from an additional 20 natural lakes. The EPA–NES provided data from 193 natural lakes and 433 artificial lakes located throughout the United States. Because some natural lakes had more than 1 year of data, there are 19 lakes represented more than once in the data set. One lake is represented 4 times, 7 lakes are represented 3 times, and 11 lakes are represented twice. Thus we have 723 observations on 704 lakes. This large sample includes a wide range of limnological conditions ranging from oligotrophic to eutrophic (Table 1). Some lakes are stratified, and some are not. Areal phosphorus loading rates range from 30 to 820 000 $\mathrm{mg \cdot m^{-2} \cdot yr^{-1}}$. Total phosphorus concentrations range from 4 to 2600 $\mathrm{mg \cdot m^{-3}}$. Lake mean depths range from 0.2 to 307 m. Hydraulic flushing rates range from 0.001 to 1800 $\mathrm{yr^{-1}}$, and calculated phosphorus sedimentation coefficients range from −290 to 490 $\mathrm{yr^{-1}}$.

The data were examined for obvious errors, but we made no judgments on the quality of the data. Indeed, this sample includes some lakes that other authors excluded in formulating their models. Obviously, the quality of the data is variable. Investigators used different methods to collect data, and some lakes were studied intensively while other lakes were sampled only a few times during extensive lake surveys. In the procedure to estimate phosphorus sedimentation coefficients, all the errors involved in estimating phosphorus loading, lake mean depth, hydraulic flushing rate, and total phosphorus concentration are incorporated into the sedimentation coefficient. Negative phosphorus sedimentation coefficients might result from the combined effects of the errors of estimation or indicate lakes that are not in a steady state. Because the data range over orders of magnitude, we expected that even occasional large errors would not obscure general relationships. Empirical phosphorus loading models that can predict total phosphorus concentration in this large sample of lakes would be of great practical value in applied problems where less than ideal data often are available.

To avoid the problems associated with developing and testing a model with the same data, we randomly sorted the lakes into two data sets. One data set (model development), which included 151 natural and 210 artificial lakes, was used to determine the limnological factors that influence phosphorus sedimentation rates. The other data set (model verification), which included 139 natural and 233 artificial lakes, was used to test the predictive abilities of the empirical models. Because the values of most parameters spanned several orders of magnitude and it was reasonable to assume that variances were proportional to means, all data values were transformed to their natural logarithms prior to statistical analyses unless stated otherwise.

TABLE 1. Mean values and related statistics for annual areal phosphorus loading rates ($\mathrm{mg \cdot m^{-2} \cdot yr^{-1}}$), total phosphorus concentrations ($\mathrm{mg \cdot m^{-3}}$), mean depths (m), hydraulic flushing rates ($\mathrm{yr^{-1}}$), and calculated sedimentation coefficients ($\mathrm{yr^{-1}}$) for 723 natural and artificial lakes.

Variable	Lake type	No. in sample	Mean ± SD	Range
Areal phosphorus loading (L)	Natural	290	2800±8500	30–76 000
	Artificial	433	15000±62000	40–820 000
Total phosphorus (TP)	Natural	290	120±250	4–2600
	Artificial	433	78±117	6–1500
Mean depth (z)	Natural	290	13±31	0.2–307
	Artificial	433	9±8	0.6–59
Hydraulic flushing rate (ρ)	Natural	290	5±14	0.001–183
	Artificial	433	35±130	0.019–1800
Sedimentation coefficient (σ)	Natural	290	24±63	−26–45
	Artificial	433	145±55	−290–490

Phosphorus Sedimentation Coefficients

To determine whether to base our model on the phosphorus sedimentation coefficient (σ) or the phosphorus retention coefficient (R), we developed a correlation matrix using σ, $\ln(\sigma)$, R, and $\ln(R)$ versus a large number of parameters including z, $\ln(z)$, q_s, $\ln(q_s)$, ρ, $\ln(\rho)$, L, $\ln(L)$, TP, and $\ln(TP)$. In every case higher correlation coefficients were found with $\ln(\sigma)$ than with σ, R, or $\ln(R)$. For this reason as well as the fact that the sedimentation coefficient considers phosphorus losses to the sediments independent of hydraulic losses through the outlet, we chose to use the sedimentation coefficient as our parameter for phosphorus losses.

Using the data set for model development, we examined several factors to determine if they could be statistically related to the calculated phosphorus sedimentation coefficients (Table 2). No significant correlations were found with algal densities as measured by chorophyll *a* concentrations, total phosphorus concentrations, or alkalinity. Phosphorus sedimentation coefficients were significantly correlated with lake mean depth ($r = -0.44$; $P < 0.01$) and areal water loading ($r = 0.65$; $P < 0.01$) as suggested by the studies of Vollenweider (1975), Kirchner and Dillon (1975), and Chapra (1975). A stronger correlation was found with hydraulic flushing rates ($r = 0.76$; $P < 0.01$), in agreement with Larsen and Mercier (1976). We also found equally good correlations for areal phosphorus loading rates ($r = 0.76$; $P < 0.01$) and the volumetric phosphorus loading rates ($r = 0.78$; $P < 0.01$) where the volumetric phosphorus loading was calculated as the annual phosphorus input divided by

TABLE 2. Correlation coefficients (r) between various limnological parameters and sedimentation coefficients. Logarithmic transformations are used.

Parameter	Natural lakes	Artificial lakes	Both combined
Volumetric phosphorus loading	0.75	0.76	0.78
Areal phosphorus loading	0.66	0.76	0.76
Hydraulic flushing rate	0.78	0.70	0.76
Areal water loading	0.55	0.63	0.65
Mean depth	−0.57	−0.37	−0.44
Total phosphorus	0.23	0.13	0.19
Chlorophyll *a*	0.19	0.02	0.05
Total alkalinity	−0.02	−0.16	−0.14

TABLE 3. Correlations (r) between the logarithms of the variables used in the phosphorus loading models.

	L	L/z	ρ	σ	z
L	1.00	0.91	0.82	0.76	−0.28
L/z		1.00	0.86	0.78	−0.66
ρ			1.00	0.76	−0.50
σ				1.00	−0.44
z					1.00

the lake volume.

Although this type of statistical analysis indicates that phosphorus sedimentation coefficients are closely correlated with some measure of water or phosphorus loading rates, it does not directly indicate a cause-and-effect relationship or which variable should be used in an empirical model. Because the water and phosphorus loading variables are closely correlated with each other (Table 3), either one could influence phosphorus sedimentation, or there could be an important unmeasured variable that also is correlated with either water or phosphorus inputs.

The positive relationships between phosphorus sedimentation coefficients and hydraulic flushing rates found in this and other studies is puzzling inasmuch as it is difficult to envision how a greater hydraulic flushing rate could increase the loss rate coefficient of phosphorus to the sediments. One might expect that a higher hydraulic flushing rate would reduce the opportunity for phosphorus to be removed by sedimentation rather than enhance it. We are led to believe that some material brought in with the water, rather than water loading itself, is responsible.

Phosphorus itself could be that material. Other things being equal, a higher phosphorus loading rate leads to greater phosphorus concentrations in the lake water. This could promote algal growth, which would provide a greater supply of sedimenting particles to remove a greater fraction of the phosphorus to the sediments. The lack of a significant correlation between phosphorus sedimentation coefficients and chlorophyll levels, however, would suggest that this hypothesis should be rejected. Because algal growth can cause calcium carbonate precipitation, and the coprecipitation of phosphorus with calcium carbonate has been suggested as an important mechanism for the removal of phosphorus from some lakes (Otsuki and Wetzel 1972; White 1974), we had expected a positive relationship with alkalinity inasmuch as it might be an index of calcium carbonate precipitation. This mechanism, however, also does not seem to be an important determinant of phosphorus sedimentation coefficients over a broad range of lakes (Table 2). Indeed, the lack of a significant correlation between total phosphorus concentrations and phosphorus sedimentation coefficients suggests that phosphorus is not the material controlling the loss of phosphorus to the sediments either directly or indirectly.

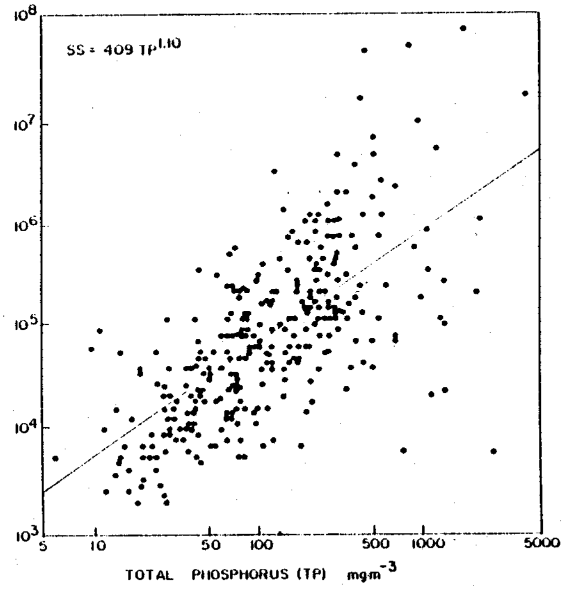

FIG. 1. Mean annual total phosphorus concentrations and mean annual suspended sediment concentrations from 301 United States rivers (U.S. Geological Survey 1977).

Jones and Bachmann (1978) suggested that allochthonous inorganic particulate materials brought in by tributary streams could act as scavengers to remove phosphorus to the sediments. Because we lack data on sediment inputs, we cannot directly test this hypothesis, but phosphorus tends to be associated with particulate materials in aquatic environments; therefore, it is possible that phosphorus loading rates are proportional to the loading rates of particulate materials. To test this idea, we obtained data on mean annual total phosphorus concentrations and mean annual suspended sediment concentrations from 301 rivers located throughout the United States (U.S. Geological Survey 1977). These data covered all the major drainages and were based in most instances on 10 grab samples from each site (Fig. 1). A significant correlation was found between mean suspended sediment concentrations and mean total phosphorus concentrations ($r = 0.67$; $P < 0.01$), thus suggesting that sediment loading rates are correlated with phosphorus loading rates.

If phosphorus loading can be used as an index of the quantity of incoming sediments, volumetric phosphorus loading rates may be a useful index of the potential concentration of sedimenting allochthonous particles in a lake. Because phosphorus loading rates are correlated with hydraulic flushing rates, greater quantities of sediments would be expected to enter lakes with higher flushing rates. Sedimentation of inflowing sediments would seem to explain the observed correlations and provide the physical mechanism for removal of phosphorus to the sediments, although we have no field data to verify these ideas.

A laboratory experiment was conducted to simulate this process. Turbid water from the Des Moines River was collected during September 1978 and mixed in different proportions with water collected on the same date from Big Creek Lake, Iowa. The mixtures were placed in 1-L graduated cylinders and allowed to settle for 24 h at a constant temperature. We found that the greater the proportion of river water, the greater was the percentage of phosphorus sedimented to the bottom of the cylinders (Table 4). This is consistent with our finding that increasing loadings to lakes and reservoirs (as simulated by greater percentages of river water) lead to higher phosphorus sedimentation coefficients.

Because the calculated phosphorus sedimentation coefficients were most strongly correlated with volumetric phosphorus loading ($r = 0.78$; $P < 0.01$) and this parameter may be an index of the concentration of potentially settleable sediments in a lake, we decided to use the best-fit linear regression equation for this relationship to predict phosphorus sedimentation coefficients in natural and artificial lakes. The resulting equation is

$$(4) \quad \sigma = 0.129(L/z)^{0.549}.$$

However, statistically significant different regression equations could be calculated for the natural and artificial lakes. The resulting equations are

$$(5) \quad \sigma = 0.162(L/z)^{0.458} \text{ for natural lakes.}$$

$$(6) \quad \sigma = 0.114(L/z)^{0.589} \text{ for artificial lakes.}$$

By using Equation 4, or Equations 5 and 6, the general Vollenweider model (Eq. 1) should predict total phosphorus concentrations in natural and artificial lakes.

We also used our model development data set to rederive the coefficients in several other published models. A nonlinear regression program (Dixon and Brown 1979) was used to fit the constants by least squares using a pseudo-Gauss–Newton algorithm. It was not possible to rederive the Kirchner and Dillon (1975) model, because the high degree of scatter that results when R is plotted against q_s ($r = -0.14$) did not allow for the use of their graphical technique. The model proposed by Reckhow (1977) also could not be used as our wide range of values for q_s could not be handled in the exponential portion of his equation. Because the Chapra (1975) and Vollenweider (1975) techniques are mathematically equaivalent, only one was rederived.

Because our models should reflect the rapid sedimentation of particulate phosphorus carried into a lake by inflowing streams, we developed two other models. The first assumes that after a rapid, initial sedimentation of particulate phosphorus near the inlets of the tributaries a constant fraction (f) of the inflowing total phosphorus will flow to the open waters to be acted on by a constant sedimentation coefficient. The form of this equation is

$$(7) \quad TP = \frac{fL}{z(\sigma + \rho)}.$$

TABLE 4. Percentage of total phosphorus settled in 24 h in mixtures of water from the Des Moines River and Big Creek Lake.

% river water	% total phosphorus settled in 24 h
0	13
1	11
10	17
50	21
90	30
100	26

A similar model would combine the initial rapid sedimentation with a sedimentation coefficient that varies with the volumetric loading:

$$(8) \quad TP = \frac{fL}{z(a(L/z)^b + \rho)}.$$

The coefficients for both models were fitted with the nonlinear regression program of Dixon and Brown (1979). The resulting equations are given in Table 5.

Model Verification

We tested the abilities of these models and the published empirical phosphorus loading models to predict the measured total phosphorus concentrations of lakes in the model-verification data set (Table 5). Correlation coefficients and best-fit linear regression equations were calculated for the relationship between measured and calculated total phosphorus concentrations. We also determined an empirical 95% confidence interval for the calculated total phosphorus concentrations of each model by calculating the standard deviation of the mean difference between the measured total phosphorus concentrations and the calculated total phosphorus concentrations. The average error was calculated as the mean of the absolute values of the differences between the untransformed calculated and measured total phosphorus values. The percentage of error was the mean of the same differences divided by the measured values and multiplied by 100. These five measures of precision were used to evaluate the respective models.

The best empirical phosphorus loading model (Table 5) was formed by using Equations 5 and 6 to predict phosphorus sedimentation coefficients separately in the natural and artificial lakes (Fig. 2). This model had the smallest 95% confidence interval (31–288% of the calculated total phosphorus value) and the highest correlation coefficient ($r = 0.83$). Almost identical results were obtained when the total phosphorus was calculated separately for natural and artificial lakes with Equation 8 that incorporated an immediate sedimentation loss along with a sedimentation coefficient based on volumetric phosphorus loading. Only slightly less precise results were obtained with the same models when no distinction was made between natural and artificial lakes or with the models of Larsen and Mercier (1976) with rederived coefficients. Several of these models can be used with about equal success to predict total phosphorus concentrations in a broad range of natural and artificial lakes. The other models tested

TABLE 5. Comparison of calculated and measured total phosphorus concentrations for the model verification data set using several proposed prediction models. Error estimates include the slope of the regression line (S), average error (AE), percentage of error (PE), and 95% confidence limits as percentages of the calculated total phosphorus value (CL). See text for methods of calculation.

Model	Correlation coefficient r	Error estimates			
		S	AE	PE	CL
This study with $\sigma = 0.162(L/z)^{0.458}$ for natural lakes and $\sigma = 0.114(L/z)^{0.589}$ for artificial lakes	0.83	0.98	38	44	31–288
This study with $TP = \frac{0.8L}{z(0.0942(L/z)^{0.422} + \rho)}$ for natural lakes and $TP = \frac{0.8L}{z(0.0569(L/z)^{0.639} + \rho)}$ for artificial lakes	0.82	0.96	38	48	31–300
This study with $\sigma = 0.129(L/z)^{0.549}$	0.81	0.97	42	46	29–301
This study with $TP = \frac{0.49L}{z(0.0926(L/z)^{0.510} + \rho)}$	0.81	0.91	42	53	31–333
[a]Larsen and Mercier (1976) with $R = \frac{1}{(1 + 0.747\rho^{0.507})}$ for natural lakes and $R = \frac{1}{(1 + 0.515\rho^{0.551})}$ for artificial lakes	0.80	0.91	42	64	25–391
[a]Larsen and Mercier (1976) with $R = \frac{1}{(1 + 0.614\rho^{0.491})}$	0.79	0.71	48	69	25–395
This study with $TP = \frac{0.603L}{z(0.257 + \rho)}$	0.77	0.71	52	78	26–435
Larsen and Mercier (1976) with $R = \frac{1}{(1 + \rho^{0.5})}$	0.79	0.72	57	92	15–500
[a]Jones and Bachmann (1976) with $\sigma = 0.94$	0.79	0.67	57	85	29–514
Jones and Bachmann (1976) with $\sigma = 0.65$	0.79	0.69	63	100	34–564
[a]Reckhow (1979) $\nu = 2.99 + 1.7q_s$	0.73	0.71	51	86	23–448
Kirchner and Dillon (1975) $R = 0.426\exp(-0.271q_s)$ $+0.574\exp(-0.00949q_s)$	0.71	0.66	53	75	19–444
Reckhow (1979) with $\nu = 11.6 + 1.2q_s$	0.68	0.63	52	71	15–440
[a]Chapra (1975) with $\nu = 5.3$	0.71	0.68	61	106	27–599
Vollenweider (1975) $\sigma = 10/z$	0.68	0.62	57	83	18–524
Chapra (1975), Chapra and Tarapchak (1976) $\nu = 12.4$	0.66	0.60	56	78	15–500

[a] Coefficients recalculated using our model development data set.

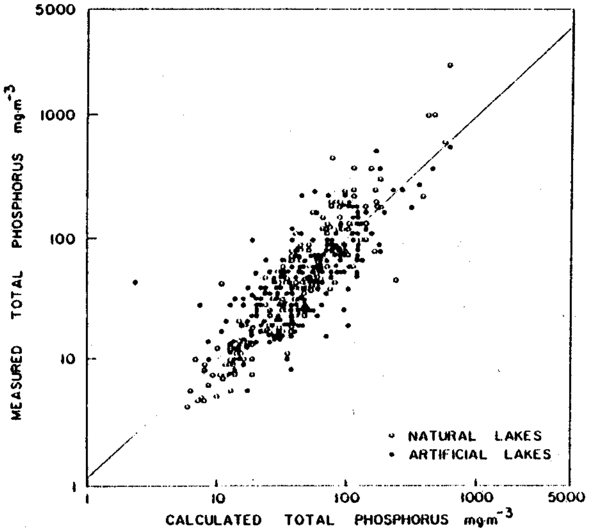

FIG. 2. Relationship between measured total phosphorus and total phosphorus calculated with equations 1, 5, and 6 of this study. The best-fit linear regression line is shown.

were less precise as indicated by the various error terms and confidence limits.

The success of the new models supports the proposed role of allochthonous particles in phosphorus sedimentation losses, but new field studies will be necessary to provide direct proof. No indication is given about the relative importance of immediate sedimentation of particulate phosphorus versus sedimentation in the open waters because the equations with and without the immediate sedimentation terms gave similar results.

Prediction of Trophic States

If we assume that these models can be used to provide a first approximation of the concentration of total phosphorus in natural and artificial lakes, can we predict other measures of lake trophic state? Chlorophyll *a* concentrations and Secchi disc transparencies often are used as indices of lake trophic state; therefore, we examined the phosphorus–chlorophyll *a* and chlorophyll *a*–Secchi relationships for the EPA-NES natural and artificial lakes. Figure 3 illustrates the phosphorus–chlorophyll *a* relationship for natural and artificial lakes. The relationship for natural lakes was not as strong as the relationships reported in the literature (Dillon and Rigler 1974a; Jones and Bachmann 1976), but the EPA-NES chlorophylls were not peak summer chlorophyll values. Most of the scatter occurs at higher phosphorus concentrations, thus suggesting that factors other than phosphorus are limiting algal levels in lakes with high phosphorus concentrations. Figure 3B clearly illustrates that the phosphorus–chlorophyll *a* relationship is much weaker (r = 0.57) in artificial lakes, with many points falling below the line previously established for natural lakes. Factors other than phosphorus would seem to be limiting algal levels in many artificial lakes, but many artificial lakes have algal levels in agreement with the phosphorus–chlorophyll *a* relationship.

Water clarity in natural lakes often is a function of algal levels (Bachmann and Jones 1974; Dillon and Rigler 1975). In the EPA-NES lakes we found a strong correlation between lake transparencies as measured by use of a Secchi disc and chlorophyll *a* concentrations (Fig. 4A; r = 0.83), but the relationship in the artificial lakes was much weaker (Fig. 4B; r = 0.44). This evidence provides quantitative support for the common observation that nonalgal turbidities are important as determinants of water clarity in many artificial lakes. From these results, it is evident that predictions of lake trophic states by using the phosphorus–chlorophyll *a* relationship or the chlorophyll *a*–Secchi relationship developed from data on natural lakes will be less reliable for artificial lakes than for natural lakes.

Discussion

Simple empirical phosphorus loading models can be useful lake management and research tools if the limitations of the models are recognized. Because the empirical models in this study were developed from data on lakes covering a broad geographic area and a wide range of limnological conditions, the models should be applicable to most natural and artificial lakes within the United States and adjacent areas of Canada. The models should be applied with caution, however, to lakes in closed basins, bays within lakes, or any lake that has characteristics different from the lakes used to develop these models. The models probably will work equally well throughout the temperate region, but they should be carefully tested before being used in other areas of the world.

We have shown that by using volumetric phosphorus loading to estimate sedimentation coefficients, we could calculate total phosphorus concentrations in both natural and artificial lakes with smaller confidence intervals than could be done with previously available models. The confidence interval of our best model, however, ranges between 31 and 288% of the calculated total phosphorus value. Regardless of the specific model, the empirical phosphorus models should be used only for making order of magnitude estimates of phosphorus concentrations in lakes because of the uncertainty associated with the predictions.

As with any type of model, the predicted results should be evaluated carefully. Special caution is warranted when the empirical models are used to predict the response of lakes to nutrient reductions. Though the models may predict a significant drop in lake phosphorus concentrations with a reduction of phosphorus loading, observed phosphorus concentrations may not drop. Previous studies have shown that lakes that were eutrophic for only a few years recovered nearly immediately with reductions in phosphorus inputs (Michalski and Conroy 1973; Schindler 1975). Lakes with a long history of high phosphorus loadings, however, recovered very slowly (Ahlgren 1972; Bjork 1972; Larsen et al. 1975). Schindler (1976) suggested that this difference in response time may be related to the degree to which sediments are saturated with nutrients. He suggested that, in lakes receiving many years of high phosphorus loading, the sediments may release phosphorus to the water for long periods, thus delaying recovery. Until research clarifies the role of bottom sediments, caution must be used when predicting the response of lakes to reductions in historic loading levels.

We believe that the empirical models developed in this

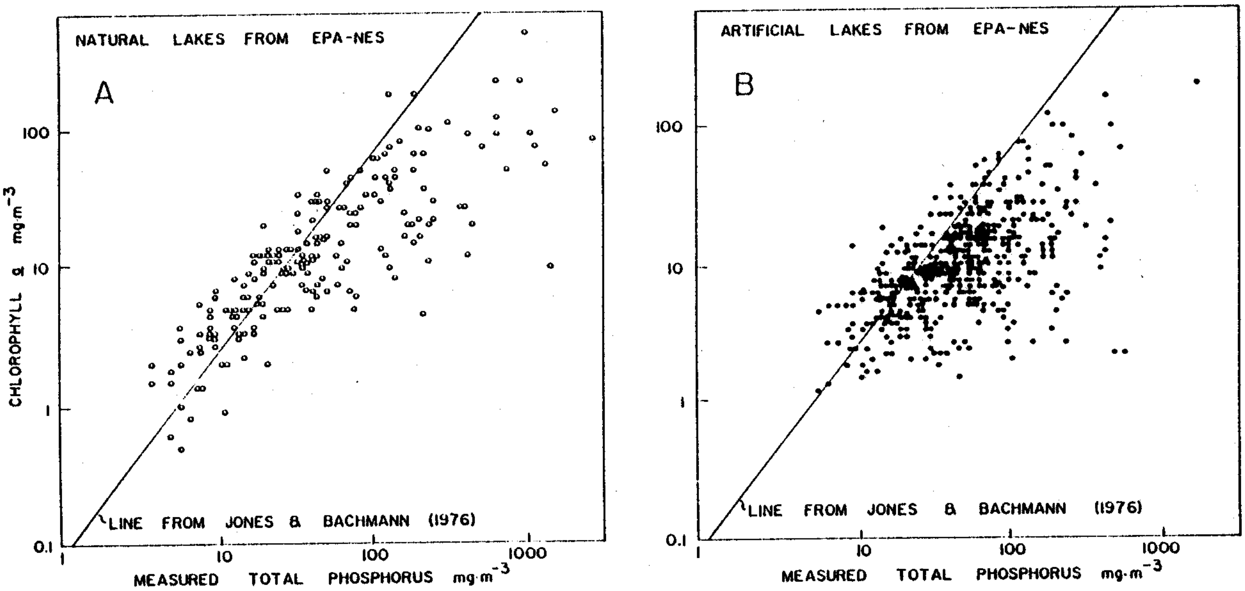

FIG. 3. Relationship between chlorophyll *a* concentrations and measured total phosphorus concentrations for the EPA–NES natural and artificial lakes.

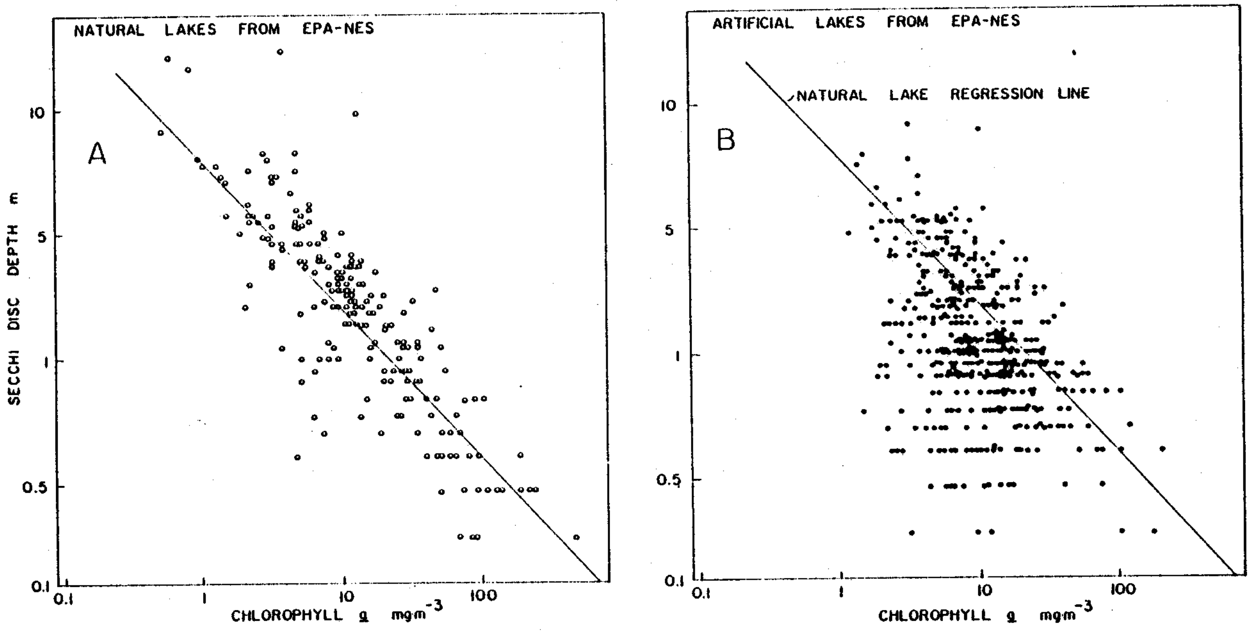

FIG. 4. Relationship between Secchi disc transparancy and chlorphyll *a* concentrations for the EPA–NES natural and artificial lakes.

study represent the limit of the predictive abilities of the simple empirical phosphorus models. The addition of more lakes to this large sample probably will not significantly alter our results. We recognize that many of the assumptions of the simple input–output models do not always hold and that there are errors, occasionally large errors, associated with the measurement of the basic data, but significant reductions in the remaining error term probably will be accomplished only when additional variables are added to the models. Although increasing the complexity of the models will be necessary if we are to model lake systems accurately, we should try to choose variables that reduce the remaining error term yet preserve the generality of the models. Although each lake probably is different, we believe that there are other general relationships that will permit us to model a wide range of lakes with reasonable accuracy.

Our studies suggest that information on sediment dynamics in a broad range of natural and artificial lakes may provide useful information for the development of better predictive models. Though phosphorus sedimentation rates could not be statistically related to total phosphorus concentrations, alkalinity, or chlorophyll *a* concentrations, our studies suggested that inputs of allochthonous sediments may be an important determinant of sedimentation rates. The inflowing sediments may absorb phosphorus from the water, remove the phosphorus to the sediments, and retain the phosphorus in the sediments similar to alum treatments used in nutrient-inactivation experiments. Even if the sediments do not settle to the bottom, they can reduce light availability to algae, thus preventing the development of high densities of plankton algae. Studies on the effects of sediments on phosphorus sedimentation rates, and on nutrient and light availability are needed.

Natural and artificial lakes often are considered as two distinct lake types. There are, of course, many obvious differences between them, but our studies indicate that these waters represe [illegible] ge of limnological conditions and should not be treated as distinctly different lake types. We were able to show that our phosphorus models could work equally well in natural or artificial lakes. Although we did find a slight improvement when we used separate equations to estimate phosphorus sedimentation rates in natural and artificial lakes, this difference may reflect some qualitative differences in the phosphorus inputs related to geographic location. The natural and artificial lakes are not randomly distributed. Many of the natural lakes in this study are located in glaciated regions while the artificial lakes are located basically in older geological formations. Differences in sediment inputs may account for the observed differences. We suggest, however, that natural and artificial lakes represent a continuum of limnological conditions that should be investigated thoroughly.

Though we can predict total phosphorus concentrations equally well in natural and artificial lakes, predictions of algal population densities as measured by chlorophyll *a* concentrations and Secchi disc transparency are less reliable in the artificial lakes. However, many artificial lakes do follow the relationships established for natural lakes. Artificial lakes that deviate may have high levels of nonalgal turbidities. Until these problems are quantified, caution should be used when predicting the algal levels and transparency of artificial lakes.

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